Convective heat transfer in a vertical Rectangular duct filled with porous matrix with Temperature dependent viscosity

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Received: 26-12-2016; Revised: 21-07-2017; Accepted: 22-07-2017

Abstract

The effect of variable viscosity on the free convective heat transfer in a vertical rectangular duct filled with porous matrix has been studied numerically. The flow is modeled using the Brinkman–Forchheimer-extended Darcy equations. One of the vertical walls of the duct is cooled to a constant temperature, while the other wall is heated to constant but different temperature. The equations governing the two dimensional steady, laminar flow and heat transfer are solved using second order finite difference method. The fluid viscosity is assumed to vary exponentially with temperature. The influence of governing parameters such as the viscosity parameter, Darcy number, inertia parameter, Grashof number, Brinkman number and aspect ratio on the velocity and temperature fields is studied. Results for the skin friction and heat transfer rate are also tabulated. The results indicate that the negative values of viscosity variation parameter show intense velocity contour in the lower half region of the duct whereas positive values of viscosity variation parameter show the intense velocity contours in the upper half region of the duct. The velocity and temperature increases with the increase in the Darcy number, Grashof number, Brinkman number in the upward and downward directions whereas it decreases with inertial parameter. The increase in aspect ratio results in the flattening of the flow contours for all values of viscosity variation parameter.

Keywords: Variable viscosity, Vertical rectangular duct, Brinkman–Forchheimer-extended Darcy model, Finite difference method.

1. Introduction

The heat and fluid flow in fluid-saturated porous media is an important problem in engineering and it has gained significant attention over the last decades. Applications of porous media can be found in grain storage,
chemical catalytic reactors, geophysical problems, solar collectors, heat exchangers, etc. These applications are reviewed in recent books by Nield and Bejan [1], Vafai [2], and Ingham and Pop [3]. Natural convection in a rectangular/square enclosure filled with a fluid-saturated porous medium under different temperature or heat flux boundary conditions has been extensively analyzed in earlier studies by Bejan [4], Prasad and Kulacki [5], Goyeau et al. [6], Manole and Lage [7], etc. In most of these studies, isothermal or isoflux thermal boundary conditions were applied to the side walls of the rectangular/square enclosures. However, non-isothermal thermal boundary conditions, which vary with time, can be found in electrical arc furnaces and rotary burners, as given by Gogus et al. [8]. Another application of this boundary condition occurred in case of cylindrical heater, as given by Saeid [9]. Hossain and Wilson [10] performed a numerical study to investigate the natural convection in a porous enclosure with internal heat generation under linearly varying temperature boundary condition.

The Darcy model, which assumes proportionality between the pressure gradient and the velocity, has been broadly employed to study a number of interesting fluids and heat transfer problems associated with heated bodies embedded in fluid saturated porous media. The model, however, is valid only for slow flows through porous media with low permeability (see Nayakama et al., [11]). At higher flow rates or in highly porous media there is a departure from the linear law and inertial effects become important. In terms of the Reynolds number based on a typical particle diameter (say), it has been found that the flow becomes non-Darcian when the Reynolds number exceeds unity (see Bear, [12]). Physically, this departure is believed to be on account of flow separation within the medium, while mathematically it manifests itself as a nonlinear term in the velocity-pressure-gradient relationship. Muskat [13] added a velocity square term, known as Forchheimer term, to account for the porous inertia effect on the pressure drop, while Brinkman [14] introduced a viscous diffusion term to consider the boundary frictional drag on the impermeable wall.

In all the above studies, the viscosity of the fluid was assumed to be uniform throughout the flow regime. It is well known that the viscosity of the most Newtonian fluids depends markedly on the temperature. This dependence, which is already quite strong in relatively inviscid liquids like water, becomes even more marked in the relatively viscous liquids for example oils.

Accordingly Garry et al. [15] and Mehta and Sood [16] have concluded that compared to the constant viscosity case, the flow characteristics substantially change when this effect is included. A local non similarity method has been used by Kafoussius and Williams [17] and Kafoussius and Rees [18] to investigate the effect of the temperature dependent viscosity on the mixed convection flow past a vertical flat plate in the region near the leading edge. From these studies, they came to a conclusion that the effect of temperature-dependent viscosity...
has to be taken into consideration whenever the viscosity of the fluid is sensitive to temperature variations. Otherwise considerable errors may occur in the characteristics of the heat transfer process. Hossain and Munir [19] investigated the mixed convective flow from a vertical flat plate. They treated the viscosity of the fluid to be inversely proportional to a linear function of temperature.

Klemp et al. [20] studied the effect of temperature dependent viscosity on the entrance flow in a channel in the hydrodynamic case. Attia and Kotb [21] studied the steady MHD fully developed flow and heat transfer between two parallel plates with temperature dependent viscosity. Later Attia [22] has extended the problem to the transient state. Attia [23] also studied the influence of temperature dependent viscosity on the MHD-channel flow of dusty fluid with heat transfer. He assumed the viscosity to vary exponentially with the temperature.

In the present work, the flow and heat transfer of viscous, incompressible, permeable fluid in a vertical rectangular duct with temperature dependent viscosity is studied. The basic governing equations are solved numerically using finite difference method.

2. **Mathematical formulation**

Figure 1 displays the schematic diagram of the two-dimensional rectangular vertical duct. The length and breadth of the duct are $a$ and $b$. It is assumed that the two sides of the duct are maintained at constant different temperatures $T_1$ at $Y=0$ and $T_2$ at $Y=b$, where $T_2 > T_1$. The other two sides of the duct are insulated, i.e. they are maintained at $\frac{\partial T}{\partial X} = 0$ at $X = a$ and at $X = 0$.

Fig. 1. Physical configuration.
This type of boundary conditions was also considered by Bejan [4], Prasad and Kulacki [5], Goyeau et al. [6], and Manole and Lage [7]. The duct is filled with fluid-saturated porous medium. It is assumed that the fluid is non-Darcian including the effect of inertial forces and taking into account the effect of viscous and Darcy dissipations. The fluid is a Newtonian incompressible fluid that appears under the laminar regime. That is, only the $Z$-component $W$ of the fluid velocity is non-vanishing. The duct walls are assumed to be rigid, impermeable and the porous medium is homogeneous and isotropic. The Oberbeck-Boussinesq approximation is supposed to hold. Fluid rises in the duct driven by buoyancy forces. Hence the flow is due to the difference in temperature and the convection sets in instantaneously. The flow is fully developed and the following relations apply here

$$U = V = 0, \quad \frac{\partial U}{\partial X} = \frac{\partial U}{\partial Y} = \frac{\partial V}{\partial X} = \frac{\partial V}{\partial Y} = 0, \quad \frac{\partial P}{\partial X} = \frac{\partial P}{\partial Y} = \frac{\partial P}{\partial Z} = 0$$

(1)

Therefore, the continuity equation gives $\frac{\partial W}{\partial Z} = 0$. One can thus conclude that $W$ does not depend on $Z$, i.e. $W = W(X,Y)$. The thermal conductivity is considered to be constant. The viscosity of the fluid is assumed to depend on the temperature. Under these assumptions and assuming thermodynamic equilibrium between porous matrix and the fluid, the equations governing the flow are (Nield and Bejan [1])

$$\frac{\partial}{\partial X} \left( \mu \frac{\partial W}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \mu \frac{\partial W}{\partial Y} \right) + g \rho \beta (T - T_0) - \frac{\mu}{\kappa} W - \frac{\mu C_p}{\sqrt{\kappa}} W^2 = 0$$

(2)

$$\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} + \frac{\mu}{K} \left[ \left( \frac{\partial W}{\partial X} \right)^2 + \left( \frac{\partial W}{\partial Y} \right)^2 \right] + \frac{\mu}{K \kappa} W^2 = 0$$

(3)

Equations (2) and (3) are solved subject to the following boundary conditions:

$$W = 0, \quad T = T_1 \quad \text{at} \quad Y = 0 \quad \text{for} \quad 0 \leq X \leq a$$

$$W = 0, \quad T = T_2 \quad \text{at} \quad Y = b \quad \text{for} \quad 0 \leq X \leq a$$

$$W = 0, \quad \frac{\partial T}{\partial X} = 0 \quad \text{at} \quad X = 0 \quad \text{for} \quad 0 \leq Y \leq b$$

(4)

$$W = 0, \quad \frac{\partial T}{\partial X} = 0 \quad \text{at} \quad X = a \quad \text{for} \quad 0 \leq Y \leq b$$

By assuming the viscosity to vary exponentially with the temperature, the viscosity takes the form $\mu = \mu_0 f(T)$ and the function $f(T)$ takes the form (Attia and Kotb [21]) $f(T) = e^{-s(T-T_1)}$, where the parameter $s$ has the dimension $[T]^{-1}$ and such that at $T = T_1$, $f(T) = 1$ and then $\mu = \mu_0$. This means that $\mu_0$ is the viscosity coefficient at $T = T_1$. 
The above equations can be converted to non-dimensional form using the following non-dimensional parameters,

\[ x = \frac{X}{b}, \quad y = \frac{Y}{b}, \quad w = \frac{W \rho b}{\mu_0}, \quad \theta = \frac{T - T_0}{T_2 - T_1}, \quad T_0 = \frac{T_1 + T_2}{2}, \quad G = \frac{g \beta \Delta T b^3 \rho_0^2}{\mu_0^2}, \quad Br = \frac{\mu_0^3}{K \Delta T \rho_0 b^2}, \quad I = \frac{C_f b}{\sqrt{\kappa}}, \quad Da = \frac{\kappa}{b^2}, \quad E = \frac{s \Delta T}{(5)} \]

The non-dimensional momentum and energy equations are written as follows.

\[ \frac{\partial w}{\partial x} \left( \frac{\partial (e^{-E_0})}{\partial x} \right) + \frac{\partial^2 w}{\partial x^2} e^{-E_0} + \frac{\partial w}{\partial y} \left( \frac{\partial (e^{-E_0})}{\partial y} \right) + \frac{\partial^2 w}{\partial y^2} e^{-E_0} + G \theta - \frac{e^{-E_0}}{Da} w - I w^2 = 0 \quad (6) \]

\[ \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + Br \int \left( \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right) \frac{Br e^{-E_0}}{Da} w^2 = 0 \quad (7) \]

The boundary conditions, used to solve the Eqs. (6) and (7) are as follows.

\[ w = 0, \quad \theta = \frac{1}{2} \text{ at } y = 0 \text{ for } 0 \leq x \leq A \]

\[ w = 0, \quad \theta = \frac{1}{2} \text{ at } y = 1 \text{ for } 0 \leq x \leq A \]

\[ w = 0, \quad \frac{\partial \theta}{\partial x} = 0 \text{ at } x = 0 \text{ and } x = A \text{ for } 0 \leq y \leq 1 \]  

3. Method of solution

The non-dimensional governing Eqs. (6) and (7) along with the boundary conditions (8) were descretized using the finite difference technique. In numerical procedure, the computational domain is divided into a uniform grid system. Both the second-derivative and the squared first-derivative terms are discretized using the central difference of second-order accuracy. The finite difference form of \( \frac{\partial^2 w}{\partial x^2} \) and \( \frac{\partial w}{\partial x} \), were discretized as

\[ \frac{\partial^2 w}{\partial x^2} = \frac{w_{i+1,j} - 2w_{i,j} + w_{i-1,j}}{\Delta x^2} + O\left(\Delta x^2\right) \quad \text{and} \quad \frac{\partial w}{\partial x} = \frac{w_{i+1,j} - w_{i-1,j}}{2\Delta x} + O\left(\Delta x^2\right), \text{ respectively.} \]

The resultant difference equations become

\[ \left( \frac{w_{i+1,j} - w_{i-1,j}}{2\Delta x} \right) - E e^{-E_0} \left( \frac{\theta_{i+1,j} - \theta_{i-1,j}}{2\Delta x} \right) + e^{-E_0} \left( \frac{w_{i+1,j} - 2w_{i,j} + w_{i-1,j}}{\Delta x^2} \right) \]

\[ + \left( \frac{w_{i+1,j} - w_{i-1,j}}{2\Delta y} \right) - E e^{-E_0} \left( \frac{\theta_{i+1,j} - \theta_{i-1,j}}{2\Delta y} \right) + e^{-E_0} \left( \frac{w_{i+1,j} - 2w_{i,j} + w_{i-1,j}}{\Delta y^2} \right) \]

\[ + G \theta_{i,j} - \sigma^2 e^{-E_0} w_{i,j} - I w_{i,j}^2 = 0 \quad (9) \]
\[
\frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{\Delta x^2} + \frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{\Delta y^2} + Br e^{-E\theta} \left[ \left( \frac{w_{i+1,j} - w_{i,j}}{2\Delta x} \right)^2 + \left( \frac{w_{i,j+1} - w_{i,j-1}}{2\Delta y} \right)^2 \right] \]  
\]  

(10)  

\[+ Br e^{-E\theta} \sigma w_{i,j}^2 = 0\]

The corresponding discretized boundary conditions become

\[w_{i,0} = -w_{i,1}, \quad \theta_{i,0} = -1 - \theta_{i,1}\]
\[w_{i,Ny+1} = -w_{i,Ny}, \quad \theta_{i,Ny+1} = 1 - \theta_{i,Ny}\]
\[w_{0,j} = -w_{1,j}, \quad \theta_{0,j} = \theta_{1,j}\]
\[w_{Nx+1,j} = -w_{Nx,j}, \quad \theta_{Nx+1,j} = \theta_{Nx,j}\]

(11)

where \(i\) and \(j\) range from 1 to \(Nx\) and 1 to \(Ny\), respectively. \(Nx\) and \(Ny\) denote the number of grids inside the computational domain in the respective \(x\) and \(y\) directions. With the given parameters \(G, Br, \phi\) and \(A\), the values of \(w_{i,j}\) and \(\theta_{i,j}\), after setting the boundary conditions (11), are iterated according to the difference Eqs. (9) and (10). Until all the values of \(w_{i,j}\) and \(\theta_{i,j}\) in the grid system are less than a prescribed tolerance, the solutions are assumed to be sought. From the image of the grid-independence study, the computational domain is divided into 100 grids in the \(y\) - direction and \(100 \times A\) grids in the \(x\) - direction. The solutions are assumed to be obtained when all elements in the grids are less than \(10^{-14}\) after a suitable number of iterations.

4. Results and Discussion

The exponential dependence of the viscosity on the temperature, results in decomposing the viscous force term in the momentum equation into two terms. The variations of these resulting terms with the viscosity parameter \(E\) and their relative magnitudes have an important effect on the flow and temperature fields. The Darcy-Brinkman-Forchrmier model is used to define the momentum and energy equation. The major parameters such as Darcy number \(Da\), inertial parameter \(I\), Grashof number \(Gr\), Brinkman number \(Br\) and aspect ratio \(A\) on the flow for positive and negative values of the viscosity parameter \(E\) is numerically evaluated and depicted graphically. To understand in a better way the flow nature, the graphs are depicted in 3-D (three dimensional), 2-D (two dimensional) and 1-D (one dimensional) for all the governing parameters. In drawing 1-D graphs the value of \(x\) is varied from 0 to 1 and the value of \(y\) is fixed as 0.5.

Figure 2a display the velocity and temperature contours in a duct for varying viscosity parameter \(E\). Under the global view of these figures, it is found that the fluid moves down ward for negative values of \(E\) and moves
upward for positive values of $E$. For constant viscosity ($E = 0$), the velocity contours look symmetric about the mid section of the duct at $y = \frac{1}{2}$. The temperature contours for variations of $E$ look similar and are nearly linear from the upper side ($y = 1$) to the lower side ($y = 0$).
Fig. 2a. Velocity and Temperature contours for different $E$ with $G = 10$, $Br = 0.5$, $A = 1$, $Da = 0.5$, $I = 2.0$.

Figure 2b is drawn at different position of $y$ for the range of $x$ between 0 and 1. The nature of velocity profiles at $y = 0.1$ and at $y = 0.9$ are reversed. In the mid section of the duct, the profiles are in the upward direction for positive values of $E$ and in the downward direction for negative values of $E$. At any position of $y$ the effect of viscosity variation parameter $E$ on the velocity field is same. One can also observe from Figure 2b that the temperature profiles for variations if $E$ is not the similar nature as that on velocity profiles. When the value of $y$ is near the left wall ($y = 0.1$) it is seen that as $E$ increases, temperature decreases for positive values of $E$ and increases for negative values of $E$ and the profile for $E = 0$ lies between $E > 0$ and $E < 0$. In the mid section of the duct ($y = 0.5$), temperature decreases for both positive and negative values of $E$. The temperature profiles for $E = 0$ have the lowest magnitude and the magnitude of temperature for $E > 0$ is more when compared to $E < 0$. When $y$ is chosen near the right wall ($y = 0.9$) the temperature increases for positive values of $E$ and decreases for negative values of $E$. The temperature profiles for different $E$ are reversed at $y = 0.9$ when compared with $y = 0.1$. This is the similar result observed by Attia [23].
Figure 3a shows the velocity and temperature counters in a duct for varying Darcy number $Da$ and viscosity variation parameter $E$. It is seen from this figure that, increasing the Darcy number increases the velocity field. For negative values of $E$ the velocity field increases at the lower half region $\left(0 < y < \frac{1}{2}\right)$ whereas for positive values of $E$, the velocity field increases in the upper half region. It is also evident from 2-D graphs that the number of contours is less in the upper half region for $E < 0$ and in the lower half region for $E > 0$. The corresponding temperature profiles shows nearly linear distribution along the $y$ direction for increasing values of Darcy number for both $E > 0$ and $E < 0$.
Fig. 3a. Velocity and Temperature contours for different E and Da with $G = 10$, $Br = 0.5$, $I = 2.0$, $A = 1$. 
Figure 3b are the pictures of 1-D for the effect of Darcy number on the flow field for both positive and negative values of $E$ and $E=0$ at $y=0.1$. As $Da$ increases, velocity decreases in the downward direction for negative values of $E$ increases in the upward direction for positive values of $E$. The temperature profiles shows that the magnitude is large for greater values of $Da$ and small for smaller values of $Da$. The velocity and temperature profiles for $E=0$ lies in between $E>0$ and $E<0$. This result is due to the fact that small values of Darcy number corresponds to densely packed porous medium and hence flow rate will be less. The effect of Darcy number is similar to the results of Umavathi [24] for constant viscosity.

![Figure 3b](image)

**Fig. 3b.** Velocity and Temperature profiles for different $E$ and $Da$ with $G=10$, $Br=0.5$, $I=2.0$, $A=1$ at $y=0.1$.

In Figure 4a, the velocity and temperature contours for varying inertial parameter $I$ are presented. It is observed from 3-D graphs that, as the inertial parameter $I$ increases, velocity decreases in both the upward and downward direction. The temperature contours are nearly linear as Brinkman number is taken as 0.5 and Darcy number is taken as 0.5. In order to identify clearly the effects of viscosity variation parameter $E$, the values of Brinkman number and Darcy number are taken less and also the inertial parameter is taken as 10. From 2-D graphs, one can reveal that for negative values of $E$ the velocity contours are less in the upper half region and for positive values of $E$ the velocity contours are less in the lower half region of the duct for all values of $I$.
Fig. 4a. Velocity and Temperature contours for different $E$ and $I$ with $G=10$, $Da=0.5$, $Br=0.5$, $A=1$.
Figure 4b are the plots of 1-D near the left plate (\(y = 0.1\)) which clearly indicates that there is no much effect of inertial parameter \(I\) on the velocity field for all values of \(E\). However, the temperature decreases in the downward direction as \(I\) increases for all values of \(E\). It is well known that the effect of drag dominates as the flow is developing. Further, the reduction in the flow rate for the effects of \(I\) is due to the fact that, apart from the frictional drag resistance due to \(Da\), in addition the inertial effects add on this resistance mechanism which further reduces the flow.

![Graph showing velocity and temperature profiles for different E and I](image)

**Fig. 4b.** Velocity and Temperature profiles for different \(E\) and \(I\) with \(G = 10\), \(Da = 0.5\), \(Br = 0.5\), \(A = 1\) at \(y=0.1\).

The effect of Grashof number \(G\) on the flow contour for variations of variable viscosity parameter \(E\) is shown in Figure 5a in a square duct \((A=1)\). It is seen from Figure 5a that for both positive and negative values of \(E\), the effect of Grashof number is to increase the velocity in the upward and downward directions. The temperature contours for small values of \(G\) show nearly linear nature and as Grashof number increases it becomes nonlinear especially in the mid section of the duct. This is due to the fact that, physically an increase in the value of \(G\) means an increase of buoyancy force which intern increases the temperature fields and hence the flow is enhanced. Here also the velocity contours are less in the lower half region for \(E < 0\) and in the upper half region for \(E > 0\).
Fig. 5a. Velocity and Temperature contours for different E and G with $Da = 0.5$, $Br = 0.5$, $I = 2.0$, $A = 1$
From 1-D graph it is clear that as Grashof number increases the magnitude of the flow increases as seen in Figure 5b.

**Fig. 5b.** Velocity and Temperature profiles for different E and G with $Da = 0.5$, $Br = 0.5$, $I = 2.0$, $A = 1$ at $y=0.1$.

The effect of Brinkman number $Br$ on the velocity and temperature contours is displayed in Figure 6a. As the Brinkman number increases, flow increases in the upward and downward directions for both positive and negative values of viscosity variation parameter $E$. The temperature contours are nearly linear in the absence and in the presence of Brinkman number. The increase in Brinkman number increases the velocity contours. This is due to fact that as Brinkman increases, the viscous dissipation effect is enhanced which results in the increase of buoyancy force. The velocity contours are less in the upper half region for $E < 0$ and in the lower half region for $E > 0$. 
Fig. 6a. Velocity and Temperature contours for different E and Br with $G = 10$, $Da = 0.5$, $I = 2.0$, $A = 1$
The effect of Brinkman number near the left wall ($y = 0.1$) is shown in Figure 6b which clearly indicate that the magnitude of velocity and temperature is more for large values of Brinkman number.

**Fig. 6b.** Velocity and Temperature profiles for different $E$ and $Br$ with $G = 10$, $Da = 0.5$, $I = 2.0$, $A = 1$ at $y = 0.1$.

The effect of aspect ratio $A$ on the flow contours are shown in Figures 7a and 8a for $A = 0.5$ and 2 respectively for variations of viscosity variation parameter $E$. It is observed from Figure 7a that the velocity contours become flat as $A$ increases for both negative and positive values of $E$. This is due to the fact that geometrically increase in the aspect ratio implies that the duct is narrow for $A < 0$ and wide for $A > 0$. Therefore the velocity contours are narrower for $A = 0.5$ and flat for $A = 2$. Here also the velocity contours are less in the upper half region for $E < 0$ and in the lower half region of the duct for $E > 0$ for any value of $A$. The temperature contours are nearly linear for any value of $E$ and $A$. 
Fig. 7a. Velocity and Temperature contours for different E with \( G = 10, \ Da = 0.5, \ Br = 0.5, \ I = 2.0, \ A = 0.5 \).

Fig. 8a. Velocity and Temperature contours for different E with \( G = 10, \ Da = 0.5, \ Br = 0.5, \ I = 2.0, \ A = 2.0 \).
To further understand the nature of the flow, the 1-D graphs are drawn and are shown in Figures 7b and 8b for \( A = 0.5 \) and 2 respectively. Here also it is very clear that the velocity and temperature profiles are narrower for \( A = 0.5 \) and become flat for \( A = 2 \). The profile for \( E = 0 \) lies in between \( E > 0 \) and \( E < 0 \).

**Fig. 7b.** Velocity and Temperature profiles for different \( E \) with \( G = 10, \ Da = 0.5, \ Br = 0.5, \ I = 2.0, \ A=0.5 \) at \( y=0.1 \).

**Fig. 8b.** Velocity and Temperature profiles for different \( E \) with \( G = 10, \ Da = 0.5, \ Br = 0.5, \ I = 2.0, \ A=2.0 \) at \( y=0.1 \).
The volumetric flow rate and skin friction \( \frac{dw}{dy} \) at \( y = 0.1 \) and \( \frac{dw}{dx} \) at \( x = 0.1 \) for different values of viscosity variation parameter is shown in Tables 1a and 1b. The volumetric flow rate increases as \( E \) increases for all the parameters such as Darcy number, inertial parameter, Grashof number, Brinkman number and aspect ratio. The volumetric flow rate \( E = 0 \) lies in between \( E > 0 \) and \( E < 0 \). That is to say that, \( Q \) is less for \( E = -1 \) when compared with \( E = 0 \) and more for \( E = 1 \) when compared with \( E = 0 \). As \( Da, G, Br \) and \( A \) increases the volumetric flow rate increases for all values of \( E \). As the inertial parameter \( I \) increases \( Q \) decreases. The shear stress \( \frac{dw}{dy} \) at \( y = 0.1 \) shows that \( \frac{dw}{dy} \) at \( y = 0 \) decreases in magnitude as \( E \) increases whereas it increases at \( y = 1 \) for all the governing parameters. As \( Da, G \) and \( A \) increases \( \frac{dw}{dy} \) at \( y = 0 \) increases in magnitude for all values of \( E \) whereas it decreases for increasing values of \( I \) and \( Br \). \( \frac{dw}{dy} \) at \( y = 1 \) increases in magnitude with \( Da, G, Br \) and \( A \) whereas it decreases with \( I \) for all values of \( E \).

The value of shear stress \( \frac{dw}{dx} \) at \( x = 0 \) and at \( x = 1 \) are exactly the same values except in signature. Here also the shear stress increases in magnitude as \( E \) increases for all governing parameters. \( \frac{dw}{dx} \) at \( x = 0.1 \) increases with \( Da, G, Br \) and \( A \) whereas it decreases for \( I \) for all values of \( E \).

**Table 1a.** Values of volumetric flow rate and skin friction.

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<td>-0.190569</td>
<td>-0.486564</td>
<td>1.298394</td>
<td>-0.190381</td>
<td>-0.481145</td>
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<tr>
<td>( G = 1 )</td>
<td>( G = 25 )</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>-1</td>
<td>-1.243968E-3</td>
<td>-0.81997E-2</td>
<td>-1.92238E-2</td>
<td>-1.541446E-2</td>
<td>-1.139628</td>
<td>-0.504465</td>
</tr>
<tr>
<td>0</td>
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<td>-3.05742E-2</td>
<td>-3.05792E-2</td>
<td>1.508538E-2</td>
<td>-0.724874</td>
<td>-0.802730</td>
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<td>-1.92206E-2</td>
<td>-4.82084E-2</td>
<td>4.940523E-2</td>
<td>-0.452799</td>
<td>-1.274184</td>
</tr>
<tr>
<td>( Br = 0 )</td>
<td>( Br = 1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>-1.235545E-2</td>
<td>-0.4811027</td>
<td>-0.192175</td>
<td>-1.028868E-2</td>
<td>-0.472885</td>
<td>-0.195369</td>
</tr>
<tr>
<td>0</td>
<td>-7.49525E-12</td>
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<td>-0.305481</td>
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<td>-0.300582</td>
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<tr>
<td>1</td>
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<td>-0.481103</td>
<td>1.457573E-2</td>
<td>-0.188854</td>
<td>-0.489972</td>
</tr>
<tr>
<td>( A = 0.5 )</td>
<td>( A = 2.0 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The values of the rate of heat transfer is shown in Table 2 for variations of viscosity variation parameter $E$ on all the governing parameters. As $E$ increases, rate of heat transfer decreases at $y=0.1$ for all values of $E$ on all the governing parameters. The rate of heat transfer $\frac{d\theta}{dy}$ at $y=0$ increases with $Da$, $G$, $Br$ and $A$ and decreases with $I$ for all values of $E$. $\frac{d\theta}{dy}$ at $y=1$ decreases with $Da$, $G$, $Br$ and $A$ and increases with $I$ for all values of $E$.

### Table 1b. Values of skin friction.

<table>
<thead>
<tr>
<th>$E$</th>
<th>$\frac{dw}{dx}$</th>
<th>$\frac{dw}{dx}$</th>
<th>$\frac{dw}{dx}$</th>
<th>$\frac{dw}{dx}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x=0$</td>
<td>$x=1$</td>
<td>$x=0$</td>
<td>$x=1$</td>
</tr>
<tr>
<td>-1</td>
<td>5.20212899E-2</td>
<td>-5.55947089E-2</td>
<td>5.55947089E-2</td>
<td>5.55947089E-2</td>
</tr>
<tr>
<td>0</td>
<td>3.467765489E-3</td>
<td>-3.467765489E-3</td>
<td>4.20977976E-3</td>
<td>-4.20977976E-3</td>
</tr>
</tbody>
</table>

### Table 2. Values of rate of heat transfer.

<table>
<thead>
<tr>
<th>$E$</th>
<th>$\frac{d\theta}{dy}$</th>
<th>$\frac{d\theta}{dy}$</th>
<th>$\frac{d\theta}{dy}$</th>
<th>$\frac{d\theta}{dy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$y=0$</td>
<td>$y=1$</td>
<td>$y=0$</td>
<td>$y=1$</td>
</tr>
<tr>
<td></td>
<td>$Da = 0.5$</td>
<td>$Da = 5.0$</td>
<td>$Da = 0.5$</td>
<td>$Da = 5.0$</td>
</tr>
</tbody>
</table>
The validity of the present solutions in the presence of viscosity variation parameter $E$ is justified with the solutions of Umavathi [24] for constant viscosity and shown in Table 3. The values agree very well with Umavathi [24].

**Table 3.** Comparison of the present results with Umavathi [24] at $y = 0.015$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>Constant viscosity</th>
<th>Variable viscosity $E=0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Velocity</td>
<td>Temperature</td>
</tr>
<tr>
<td>0.005</td>
<td>-7.799E-4</td>
<td>-0.48474</td>
</tr>
<tr>
<td>0.105</td>
<td>-0.00709</td>
<td>-0.48472</td>
</tr>
<tr>
<td>0.205</td>
<td>-0.0092</td>
<td>-0.48469</td>
</tr>
<tr>
<td>0.305</td>
<td>-0.01013</td>
<td>-0.48467</td>
</tr>
<tr>
<td>0.405</td>
<td>-0.01053</td>
<td>-0.48465</td>
</tr>
<tr>
<td>0.505</td>
<td>-0.01064</td>
<td>-0.48465</td>
</tr>
<tr>
<td>0.605</td>
<td>-0.01051</td>
<td>-0.48465</td>
</tr>
<tr>
<td>0.705</td>
<td>-0.01007</td>
<td>-0.48467</td>
</tr>
<tr>
<td>0.805</td>
<td>-0.00906</td>
<td>-0.48469</td>
</tr>
<tr>
<td>0.905</td>
<td>-0.00675</td>
<td>-0.48472</td>
</tr>
<tr>
<td>0.995</td>
<td>-7.799E-4</td>
<td>-0.48474</td>
</tr>
</tbody>
</table>

5. Conclusion
The effects of viscosity variation parameter, Darcy number, Grashof number, inertial parameter, Brinkman number and aspect ratio on the flow and heat transfer of natural convection for a non-Darcy model in a vertical rectangular duct were discussed. The results drawn were,

1. The negative values of viscosity variation parameter show intense velocity contours in the lower half region of the duct whereas positive values of viscosity variation parameter show the intense velocity contours in the upper half region of the duct.
2. The temperature contours remain almost linear for any variations of governing parameters for all values of viscosity variation parameter.
3. The velocity and temperature increase with the Darcy number, Grashof number, Brinkman number in the upward and downward directions whereas it decreases with inertial parameter. The increase in aspect ratio results in the flatting of the contours for all values of viscosity variation parameter.
4. The rate of heat transfer at \( y = 0 \) decreases and at \( y = 1 \) increases for increasing values of Darcy number, Grashof number, Brinkman number and aspect ratio for all values of viscosity variation parameter and reversal effect was observed for inertial parameter.
5. The present results agree very well with Umavathi [24] for constant viscosity and with Attia [23] for variable viscosity.

**NOMENCLATURE**

- \( A \) aspect ratio \((b/a)\)
- \( a \) horizontal distance
- \( b \) vertical distance
- \( Br \) Brinkman number \((\mu^3/(K \Delta T \rho^2 b^2))\)
- \( Da \) Darcy number \((\kappa/b^3)\)
- \( E \) viscosity variation parameter \((s \Delta T)\)
- \( G \) Grashof number \((g \beta \Delta T b^3 / \mu^2)\)
- \( I \) inertial parameter \((C_f b / \sqrt{\kappa})\)
- \( K \) conductivity of the fluid
- \( T \) temperature
- \( T_0 \) reference temperature
- \( U, V, W \) velocity component
- \( u, v, w \) dimensionless velocity component
\[X, Y, Z\] space coordinate
\[x, y, z\] dimensionless space coordinate

**Greek symbols**

\[\rho\] density
\[\kappa\] permeability of the porous media
\[\mu\] viscosity
\[\theta\] dimensionless temperature

**Acknowledgment**

J.C. Umavathi is thankful for the financial support under the UGC-MRP F.43-66/2014 (SR) Project.

**Conflict of interest**

The author don’t have anyone as conflict of interest

**References**


